A Panorama of Data Uncertainty Models

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Roadmap

- **Information**
- **Typology of defects**
  - Subjectivity
  - Incompleteness
  - Uncertainty
- **Uncertainty models**
- **Conclusion**

Information

Information

- **What it does mean?**
  - Refers to any collection of symbols or signs produced either through
    - the observation of natural/artificial phenomena or
    - cognitive human activity
  - With a view to help an agent understand
    - the world,
    - the current situations,
    - making decisions,
    - communicating with other human or artificial agents

Information

- **Basic aspects**

Origin

Objective Information

- **Sensor measurements**

Subjective Information

- **Direct perceptions of events**
- **Uttered by individuals (testimonies)**

Form

Quantitative / Numeric

- **Numbers, intervals**
- **Functions, statistics**

Qualitative / Symbolic

- **Natural Language**
- **Logic**

Information

- **Kind of information**

Singular Information
(Data)

Refers to
- a particular situation
- a response to a question

Stated as
- An observation (A patient has fever at a given instant)
- A testimony (The killer was a man)

Can be unreliable, imperfect (imprecise, uncertain)

Generic Information
(Knowledge)

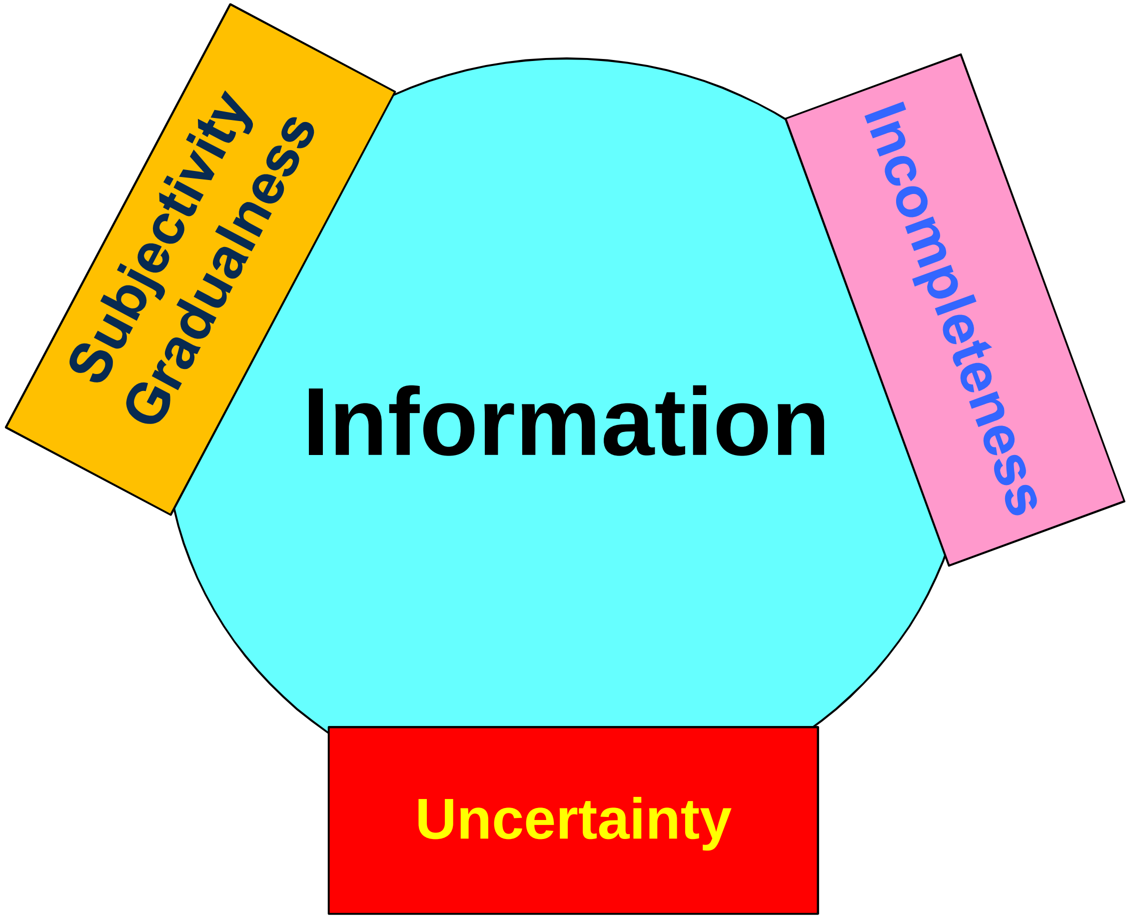
Refers to
- a collection of situations
- a population of entities

Expressed as
- Physical law, statistical model (Built from a representative sample of observations)
- Piece of commensense knowledge (Birds fly)

Presence of exceptions

A Typology of Defects

A typology of defects



Subjectivity

Subjectivity

- **Subjective information**
  - Very common for human beings
  - Inherent in the way they naturally interact with their environment
  - Depends on the person providing and interpreting the information
  - Example
    - Sensory information is subjective
    - Human agent has a different capability for seeing colours
- **Aspects of subjectivity**
  - Perception and sensory information
    - Spammer worker, Beautiful city, Nice weather, …
  - Expressions du language naturel
    - Very young person, Close to the city centre, Big building …

Subjectivity

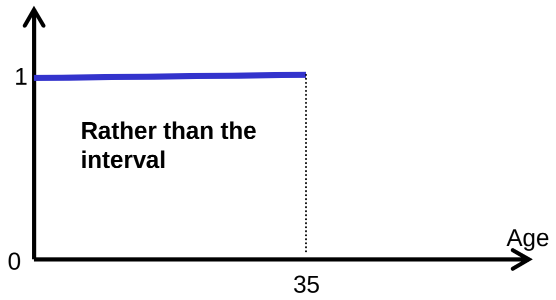
- **Nature of Subjective information**
  - Expressed either
    - Qualitatively by means of words with all the vagueness of natural language, or
    - Numerically through estimations or approximate values.
- **Subjectivity is everywhere**
  - Open sources, Blogs, Forums, Image and Video Databases, …
  - Multiple factors of subjectivity have to be dealt with
    - Data contain subjective elements in texts or images
    - User queries based on elements of natural language, may contain subjectivity

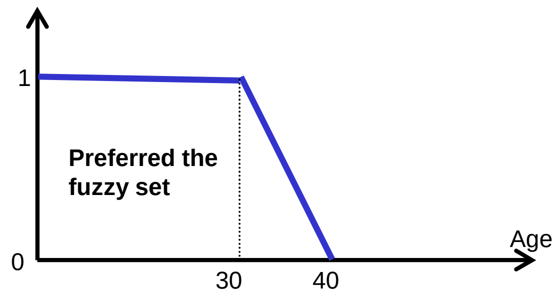
Subjectivity

- **Representation Framework**
  - **Intelligence Computational** (Soft computing, Fuzzy sets, Rough sets, …) offers appropriate approaches to manage subjectivity
    - It allows us to represent imprecise, vague, approximate or incomplete descriptions in an unified way
  - The key concept is the membership functions of fuzzy sets
    - Generalizes the idea of class to the categories with ill-defined and unclear boundaries
    - which can be shared by several people and,
    - modified to come to a consensus if necessary.

Subjectivity

- **Fuzzy-Set-Based Representation**
  - **Repesentation of Young**

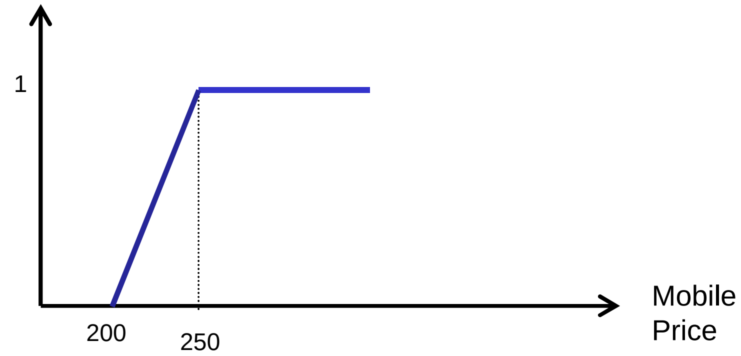




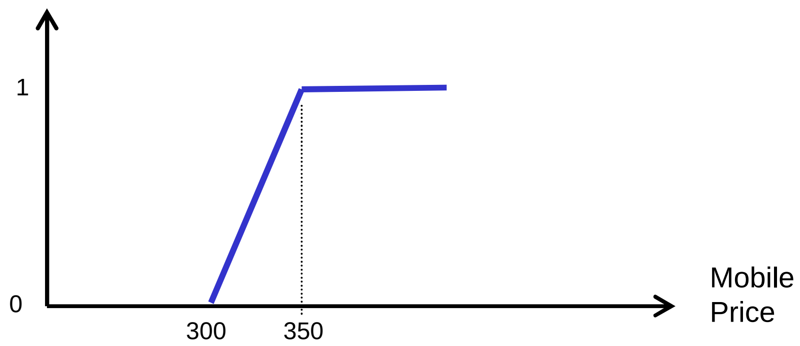
  - Advantages of the gradual representation
    - Less sensitive to the choice of thresholds
    - It is often more informative than Boolean representation : plausibility ranking between the values

Subjectivity

- **Fuzzy-Set-Based Representation**
  - Context/environment dependent: Consider the description Expensive



Student Perception



Engineer Perception

Subjectivity

- **Refine subjective information**
  - Use of **Linguistic Modifiers**
  - Let the subjective description **Young**
  - Reinforcement the meaning
    - Very Young, …
  - Weakening the meaning
    - More or less Young, …
  - Other types of modifiers
    - Fairly, slightly, moderately, ….

Subjectivity

- **Aggregation operators : A rich and large panoply**
  - Complex Subjective Categories
    - Worker less competent and fairly certain
  - Conjunctive aggregation : Triangular Norms Operators
  - Disjunctive aggregation : Co-Norms Operators
  - Compensatory aggregation : Several Variants of the Average Operator
  - Linguistic Quantifiers : Most, Almost all, Many, At least, …
    - Almost all the workers are spammers
    - Most of the answers are similar
  - Importance in Categories
    - Assigning importance to a category : Weighted aggregation
    - Assigning importance to a set of categories : Choquet/Sugeno Integrals

Incompleteness

Incompleteness

- **Meaning**
  - It is not sufficient to allow the agent to answer a relevant question in a given context
  - To not know precisely the value of a parameter
    - Imprecision is a form of incompleteness
    - Related to the content of information
  - Kind of questions: *what is the current value of some quantity v*?
    - The imprecision is not an absolute notion. It depends on the proper frame *S*
    - Let *v* denotes the age of a person
      - *S = {minor, major}*, *v = minor* is precise
      - *S = {0, 1, …, 150}* (in years), the term *minor* is imprecise, it provides incomplete information if the question of interest is to know the birth date of the person

HAA1

Incompleteness

- **Disjunctive sets**
  - Sets of values that are mutually exclusives
    - Used to represent incomplete information (expressed by imprecision)
    - v = age(Pierre) $\in$ {20, 21, 22, 23, 24, 25}, i.e., v = 20 $\vee$ 21 $\vee$ 22 $\vee$ 23 $\vee$ 24 $\vee$ 25
    - Only one value is real value
- **Conjunctive sets**
  - Represent precise piece of information
    - *v = sisters(Pierre):* the set of subset of possible names for *Pierre's sisters*
    - *v = {Marie, Sylvie}: is precise information on S =* 2^{NAMES}

Diapositive 19

HAA1 HADJ ALI Allel; 13/11/2018

HAA1

Incompleteness

- **Sets and Sets:** Do not mix up
  - **A set-valued variable X**: the set of languages a person can speak, *A = {English (and) French}*, it is conjunction of values, and a real set. *X* = *A* is precise
  - **An ill-known point-valued variable x**: *E = {English (exor) French}, it is a disjunction of values, and an epistemic set. x ∈ E is imprecise.*

Diapositive 20

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Uncertainty

Uncertainty

- **Meaning**
  - Understood as the inability to say whether
    - A proposition is true or false
    - An event will occur or not
  - Examples
    - Daily quantity of rain in Paris
    - Birth date of Brazilian President
    - Identification of car involved in an accident
  - To qualify uncertainty, one assign a token of uncertainty
    - Numerical
    - Symbolic (qualitative token): (very possible, not absolutely certain)
    - Interval

Uncertainty

- **Origins**

Variability of observed natural phenomena: randomness	**Lack of information:** incompleteness	**Conflicting testimonies/reports:** inconsistency
Dailiy quantity of rain in Paris Failure time of a machine	Birth date of Brazilian President	Identification of car involved in an accident
✓ Probabilistic answer in function of the frequency observed ✓ Repeatable events	✓ Answers are more or less perfect in function of the state of knowledge of an agent ✓ Non-repeatable events	✓ The more sources, the more likely the inconsistency
Random uncertainty	Epistemic uncertainty	

Uncertainty Models

Sets

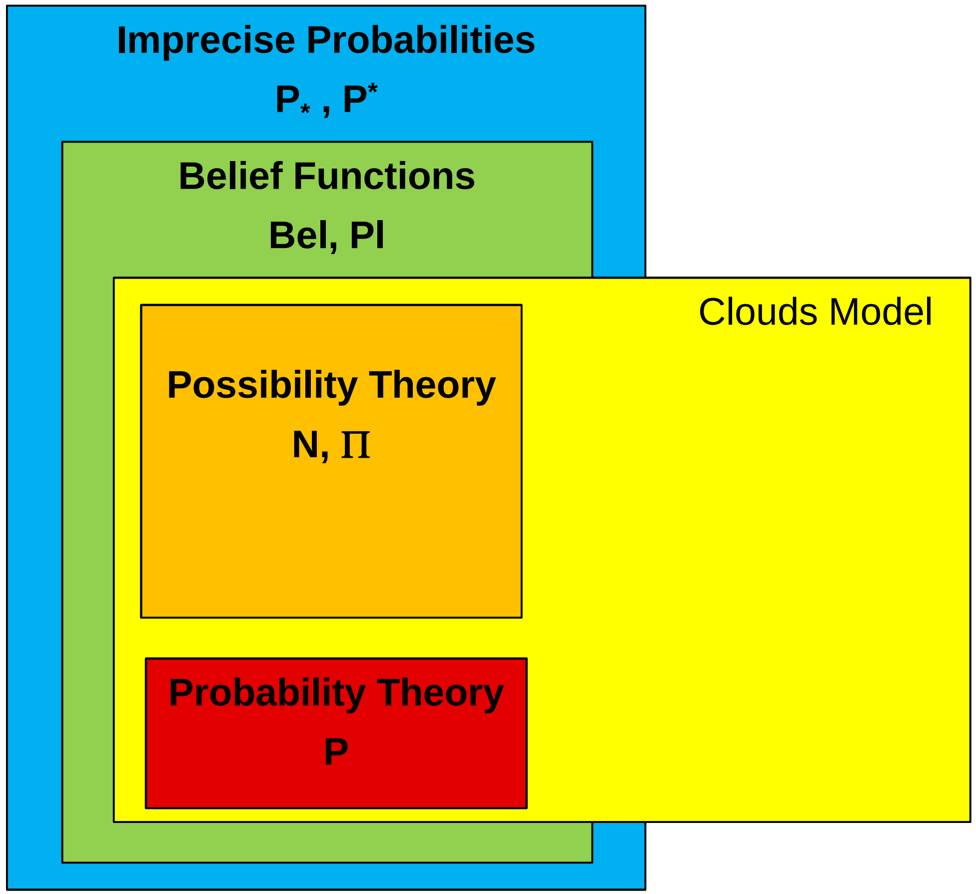
- **Sets**
  - Good for representing incomplete information, but often crude representation of uncertainty
    - Agent is not certain about the order relation r between two real parameters a1 and a2
    - He expresses $r \in \{<, =, >\}$
  - Limitations
    - The larger disjunctive set, the more uncertain relation
    - No quantification of uncertainty inherent to the available knowledge
    - Missing order between the elements of the set: not able to express that an element is more plausible than another.

Need of uncertainty models that are more informative than sets

Weighted Models

- **Weighted models**
  - v: vector of attributes relevant for the agent
  - S: domain of v (called a frame: set of all states of the world)
  - A: subset of S, called event or proposition that asserts $v \in A$
  - **Principe**
    - Assign to each event A a number $g(A)$ in the unit interval
    - $g(A)$ degree of confident of an agent in the truth of $v \in A$
  - **Natural requirements** *(of the confidence function g)*
    - $g(\emptyset) = 0$: the contradictory proposition $\emptyset$ is impossible
    - $g(S) = 1$: the tautology S is certain
    - If $A \subseteq B$ then $g(A) \leq g(B)$: monotonicity with w.r.t. inclusion *(the more imprecision a proposition, the more certain it is)*
  - **Important consequences**
    - $g(A \cap B) \leq \min(g(A), g(B))$
    - $g(A \cup B) \geq \max(g(A), g(B))$

Weighted Models



Probability theory

- The oldest among uncertainty theories
- The most widely acknowledged
- **Distribution of probability p**
  - Let $(\Omega, \mathcal{A}, P)$ a probability space
  - A probability distribution p is a non-negative mapping

$$p : \Omega \rightarrow [0, 1]$$

such that $\Sigma_{\omega \in \Omega}\, p(\omega) = 1$

- **Measure of probability P**
  - Let $A \subseteq \Omega$, an event
    - $P(A) = \Sigma_{\omega \in A}\, p(\omega)$
  - Axioms
    - $P(\emptyset) = 0$; $P(\Omega) = 1$
    - $\forall$ A, B $\subset \Omega$, if $A \cap B = \emptyset$, $P(A \cup B) = P(A) + P(B)$ (Additivity)
    - $\forall A \subset \Omega$, $P(A) = 1 - P(A^c)$, with A^c is the opposite event of A (Duality)

Auto dual model

Probability theory

Two type of interpretations

Frequentist (randomness)

- Capture variability through repeated observations
- Rely on statistical data

Need good knowledge of the (physical) phenomena

Subjective (belief)

- Describes a person's opinion
- Models unreliable evidence
- Not necessary related to statistics
- Precise probability values are difficult

Intervals are more faithful (sometimes linguistic terms)

Probability theory

- Probability theory relies on the use of a single probability distribution to represent uncertainty
- This can raise some serious problems
  - **Instability:** The same state of knowledge represented by incompatible distribution probabilities
    - Example: Extra terrestrial life

      Generally, people ignore whether there is a life or not

      $P_1(\text{Life}) = P_1(\text{Nolife}) = 1/2$ on $S_1 = \{\text{Life}, \text{Nolife}\}$

      The agent can discern between animal life (Alife) and vegetal life only (Vlife),

      with the frame $S_2 = \{\text{Alife}, \text{Vlife}, \text{Nolife}\}$, in this case the ignorance expresses

      $P_2(\text{Alife}) = P_2(\text{Vlife}) = P_2(\text{Nolife}) = 1/3$

The two representations P_1 and P_2 of ignorance are clearly inconsistent

Probability theory

- Probability theory relies on the use of a single probability distribution to represent uncertainty
- This can raise some serious problems
  - Ambiguity: No difference between uncertainty due to incomplete information and uncertainty due to randomness
    - In the dice game,
      - ✓ Agent 1 knows that the dice is unbiased

        $P(1) = \ldots = P(6) = 1/6$

        Pure randomness

      - ✓ Agent 2 ignores everything about that dice (He has not try it)

        $P(1) = \ldots = P(6) = 1/6$

        Insufficient Reason Principle

- No bijection between the possible epistemic states of the agents and probability distributions
- Problematic in dynamical framework: available information evolves

Probability theory

- **In summary probability theory**
  - ☑ Dedicated to random phenomena
  - ☑ Unable to model uncertainty due to lack of knowledge or missing information
  - ☑ Information demanding (requires to known Ω or prior probabilities)
  - ☑ A pure numeric model (very difficult in the case of subjective probabilities)
  - ☑ Complex computation and reasoning
  - ☑ Additive: error propagation and amplification
  - ☑ A single measure to represent uncertainty ($P(A)$ implies $P(A^c)$)

Probability theory

- ✓ **Probability theory is insufficient to handle all facets of uncertainty**

- ✓ **Need for a representation of uncertainty that:**
  - ✓ **is less demanding than probability theory**
  - ✓ **explicitly allows for incomplete information**
  - ✓ **is of qualitative/ordinal nature**

Possibility theory

- Possibility theory (Zadeh, 1978; Dubois & Prade, 1988) belongs to the family of uncertain modern theories
- Two conjugate measures are used to quantify uncertainty
  - The first characterizes the truth of A
  - The second characterizes the truth of A^c (complementary of A)
- It is not additive
- It can be purely ordinal or qualitative
- It is tailored to the modelling of uncertainty due to incomplete information
  - Human knowledge (incomplete, unreliable)
- It provides a good representation of total ignorance

Possibility theory

- **Possibility distribution:** Key concept
  - It represents a state of knowledge related to the state of a system
  - It models a flexible constraint that restricts the more or less possible values of a variable
  - U a referential (set of the states of the world)
  - x an ill-known variable (takes its values in U)
  - Possibility distribution π_x attached to (a variable) x

$$\pi_x : U \rightarrow L$$

L can be any scale of plausibility totally ordered (finite, often [0, 1], …)

  - **Conventions**
    - $\exists\, u \in U, \pi_x(u) = 1$, one value is totally possible for u
    - $\pi_x(u) = 0$, u is totally excluded as a value for u
    - $\pi_x(u) < \pi_x(u')$, $x = u'$ is more plausible than $x = u$

Possibility theory

- **Examples of possibility distribution**
  - **Precise information:** $x = u_0$

    $$\pi_x(u) = 1 \text{ if } x = u_0$$
    $$= 0, \text{ otherwise}$$

  - **Incomplete and clear information:** $x \in A$ ($A \subset U$, interval for instance)

    $$\pi_x(u) = 1 \text{ if } x \in A$$
    $$= 0, \text{ otherwise}$$

  - **Nuanced imprecise information:** $x \in F$ (where F expresses a vague information modelled thanks to a fuzzy set)

    $$\pi_x(u) = \mu_F(u),\ \forall\ u \in U$$

    The possibility that $x = u$ is given by $\mu_F(u)$

  - **Total ignorance**

    $\pi_x(u) = 1,\ \forall\ u \in U$ **(everything is possible)**

Possibility theory

- **Possibilistic uncertainty of an event:** Two measures
  - The degree of possibility of the event A
  - The degree of the impossibility of the opposite event A^c; called also certainty (necessity) degree
  - Assume the available knowledge (of an agent) is represented by π
  - How confident are we that $x \in A$ (an event A occurs)?
  - Degree of possibility that $x \in A$

    $$\Pi(A) = \sup_{x \in A} \pi(x)$$

    To what extent A is consistent with π (some $x \in A$ is possible)
  - Degree of certainty that $x \in A$

    $$N(A) = 1 - \Pi(A^c) = \inf_{x \in A^c} (1 - \pi(x))$$

    To what extent no element outside A is possible (to what extent π implies A)

Possibility theory

- **Boolean uncertainty of an event:** Two measures
  - Assume the available knowledge (of an agent) is $x \in E$ (interval, crisp set)
  - Degree of possibility that $x \in A$

    $\Pi(A) = 1$, if $A \cap E \neq \emptyset$

    $= 0$, otherwise

    Logical consistency (to what extent A is not incompatible with E)
  - Degree of certainty that $x \in A$

    $N(A) = 1$, if $E \subseteq A$

    $= 0$, otherwise

    Logical deduction (to what extent A is deducted from E)

Possibility theory

- **Boolean uncertainty of an event:** Two measures
  - **Example: x ∈ E = {red, green}** : Available knowledge
    - A = {blue},

      $\Pi(A) = N(A) = 0$ A is certainly false

    - A = {red, blue, green},

      $\Pi(A) = N(A) = 1$ A is certainly true

    - A = {red},

      $\Pi(A) = 1$; $N(A) = 0$ ($\Pi(A^c) = 1$)

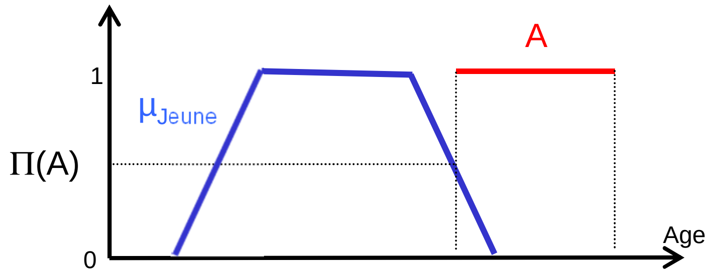
      Nothing about A (Total ignorance)

This representation is not soft, we will provide some nuance on it

Possibility theory

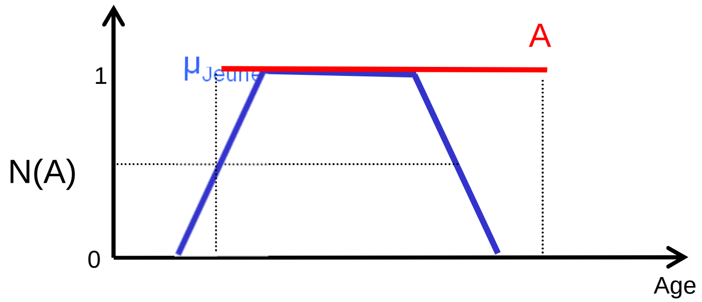
- **Gradual uncertainty of an event:** Two measures
  - Assume the available knowledge (of an agent) is $x \in F$ (Fuzzy set)
  - Degree of possibility that $x \in A$

    $$\Pi(A) = \max (\mu_F(x) \mid x \in A)$$



  - Degree of certainty that $x \in A$

    $$N(A) = \max (1 - \mu_F(x) \mid x \in A^c)$$



Possibility theory

- **Gradual uncertainty of an event:** Two measures
  - Let π a p.d. that describes the age of a person (on X = [20, 30]) :

    $\pi(20) = 0.2$, $\pi(21) = 0.3$, $\pi(22) = 0.4$, $\pi(23) = 0.6$, $\pi(24) = 0.8$, $\pi(25) = 1$,

    $\pi(26) = 0.8$, $\pi(27) = 0.6$, $\pi(28) = 0.4$, $\pi(29) = 0.3$, $\pi(30) = 0.2$

  - Let A = {22, 23, 24, 25, 26, 27}: age between 22 and 27

    $\Pi(A) = \max(\pi(22), \pi(23), \pi(24), \pi(25), \pi(26), \pi(27)) = 1$

  - Let A = {27, 28, 29, 30, 31, 32}: age between 27 and 32

    $\Pi(A) = \max(\pi(27), \pi(28), \pi(29), \pi(30), \pi(31), \pi(32)) = 0.6$

  - Now let A = {23, 24}, B = {23, 25, 27}, C = {23, 24, 25, 26, 27, 28},
    D = {20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30}.

    $\Pi(A^c) = 1$, $\Pi(B^c) = 0.8$

    $\Pi(C^c) = 0.4$, $\Pi(D^c) = 0$

Possibility theory

- **Fundamental axioms**
  - $\Pi(A \cup B) = \max(\Pi(A), \Pi(B))$
  - $N(A \cap B) = \min(N(A), N(B))$
  - $\Pi(\emptyset) = N(\emptyset) = 0$; $\Pi(U) = N(U) = 1$
  - $N(A) \leq \Pi(A)$
  - $N(A) > 0 \Rightarrow \Pi(A) = 1$
  - $\mathrm{Max}(\Pi(A), \Pi(A^c)) = 1$: one among A or non A is possible
  - $\Pi(A) = \Pi(A^c) = 1$: models total ignorance

- **Mind that most of time:**
  - $\Pi(A \cap B) < \min(\Pi(A), \Pi(B))$
  - $N(A \cup B) > \max(N(A), N(B))$
  - Example: Total ignorance on A and B = A^c

Possibility theory

- **Gradual uncertainty**
  - Uncertainty about A is represented by two evaluations (N(A), Π(A))
    - (1, 1) A is certainly true
    - (α, 1) A is somewhat certain
    - (0, 1) Total ignorance
    - (0, α) Non A is somewhat certain
    - (0, 0) A is certainly false
  - Ordinal nature of the degrees
    - N(A) > N(B) A is more certain than B
  - The degrees can be numeric or qualitative: only max/min operators are used
    - No error amplification

Possibility theory

- **Possibilistic Setting**
  - ☑ Uncertainty processing founded on the idea of order
    - ➢ Only the order is important, not the precise values of the degrees
    - ➢ The knowledge can be then encoded in a pure qualitative way
    - ➢ While the probabilistic knowledge must be numeric
  - ☑ Gradual theory of uncertainty that makes sense in ordinal structures
  - ☑ Copes with total/partial ignorance
  - ☑ More adapted to (human) incomplete information

Belief Functions Theory

- **Introduction**
  - Proposed by Shafer in 1976, also called theory of evidence
  - The advantage of the evidence theory is twofold:
    - it allows modeling both uncertainty and imprecision (due to the lack of information)
    - it represents a generalization of both probabilistic and possibilistic models

Belief Functions Theory

- **Illustrative Example**

Location of some conferences for the next year, which should be either in Europe or U.S.A.

Assume that we know the following probability distribution

<Europe, 0.5> and <U.S.A, 0.5>.

Now, if it is in Europe, it will be in Paris or London. If it is in US.A., it will be in Phoenix, Iowa City or Kansas City. But we don't know any probability distribution for these locations. It is then natural to represent this information as

<{Paris, London}, 0.5>

<{Phoenix, Iowa City, Kansas City}, 0.5>

Let A = {Paris, London}, m(A) = 0,5 is the mass belief assigned to A only and to none of its subsets.

Belief Functions Theory

- **Mass function:** Key concept
  - Let $\Omega = \{H_1, H_2, \ldots, H_3\}$ a finite set of answers to some question of interest
    - Ω : Frame of Discernment
    - H_i are mutually exclusive and exhaustive
  - **Definition**
    - Defined on sets and not on singletons
      - ✓ while probability distribution is a point theory
    - $m : 2^{\Omega} \to [0, 1]$ and satisfies $m(\emptyset) = 0$ and $\Sigma_{A \subseteq \Omega}\, m(A) = 1$
    - $m(A)$
      - ✓ belief degree assigned exactly to the hypothesis A, and to none more specific hypothesis
      - ✓ $m(\Omega)$ represents the degree of total ignorance
    - The sets A of Ω such that $m(A) > 0$ are called focal elements of m

Belief Functions Theory

- **Particular cases**
  - Total ignorance: $m(\Omega) = 1$ (vacuous mass function)
  - Certainty: $m(\{\omega\}) = 1$ for one $\omega \in \Omega$ (certain mass function)
  - Certain imprecise knowledge: $m(A) = 1$ for some $A \subseteq \Omega$, $|A| > 1$

    (categorical mass function)
  - Probabilistic uncertainty: $m(A) = 0$ for $A \subseteq \Omega$, s.t. $|A| > 1$

    (Bayesian mass function)
  - Possibilistic uncertainty: focal elements ($m(A_i) > 0$) $A_1 \subseteq A_2 \subseteq \ldots \subseteq A_n$,

    (Consonant mass function)

Belief Functions Theory

- **Example 1: A murder**
  - A murder has been committed. There are three suspects:

    Ω = {Peter, John, Mary}

  - A witness saw the murder going away, but he is short-sighted and he only saw that it was a man. We know that the witness is drunk 20% of the time.
  - This piece of evidence can be represented by
    - m({Peter, John)} = 0,8
    - m(Ω) = 0,2
  - The mass 0,2 is not committed to {Mary}, because the testimony does not accuse Mary at all!

Belief Functions Theory

- **Example 2: Presidential Election**
  - For the next presidential election, one wish to make a forecast on the winning candidate through one or more surveys:

$$\Omega = \{G_1, G_2, D_1, D_2, D_3\}$$

  - Left candidates are designed by: $G_1 \cup G_2$
  - Right candidates are designed by: $D_1 \cup D_2 \cup D_3$
  - A survey conducted in the street gives the following results:
    - $m(G_1) = 0{,}2$ $m(G_2) = 0{,}05$ $m(G_1 \cup G_2) = 0{,}1$
    - $m(D_1) = 0{,}3$ $m(D_1 \cup D_2 \cup D_3) = 0{,}1$
    - $m(\Omega) = 0{,}25$

Belief Functions Theory

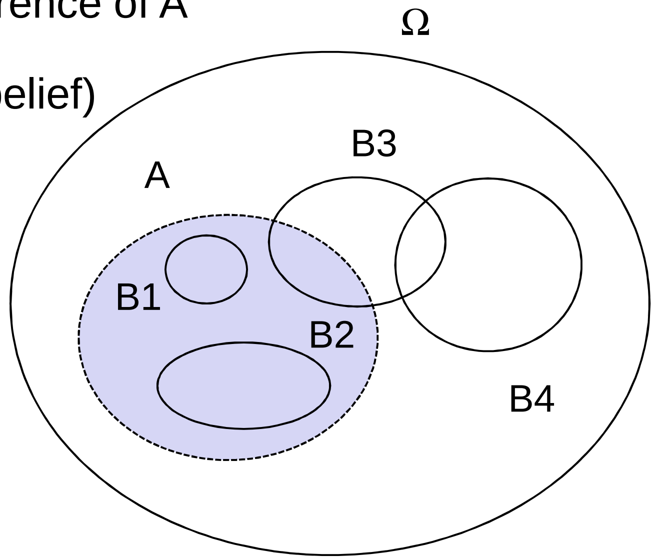
- **Example 2: Target Classification**
  - Let us consider a classification problem of air targets
  - Assume one has the following frame of discernment

$$\Omega = \{H_1, H_2, H_3\} = \{\text{Airplane, Hélicoptère, Missile}\}$$

  - A sensor signals a presence of a fairly quick target
  - The result of classification is
    - $m(\text{Avion}) = 0{,}6$
    - $m(\text{Avion} \cup \text{Missile}) = 0{,}2$
    - $m(\Omega) = 0{,}2$

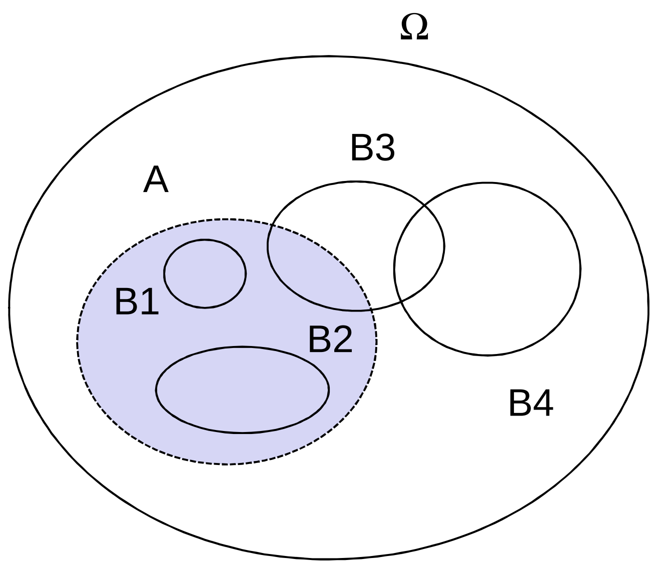
Belief Functions Theory

- **Basic set functions**
  - **Belief function Bel**
    - $Bel(A) = \Sigma_{\varnothing \neq B \subseteq A}\, m(B)$
    - Total mass of information implying the occurrence of A
    - Total part of belief that supports A (minimal belief)
  - **Example**
    - $Bel(A) = m(B1) + m(B2)$
  - **Limit conditions**
    - $Bel(\Omega) = 1$, $Bel(\varnothing) = 0$)



Belief Functions Theory

- **Basic set functions**
  - **Plausibility function Pl**
    - $Pl(A) = \Sigma_{B \cap A \neq \emptyset} m(B)$
    - Total mass of information consistent with A
    - Maximal part of belief that could support
  - **Example**
    - $Pl(A) = m(B1) + m(B2) + m(B3)$
  - **Limit conditions**
    - $Pl(\Omega) = 1, \; Pl(\emptyset) = 0)$



Belief Functions Theory

- **Basic Properties**
  - **Belief-Plausibility Relation**
    - $Pl(A) = 1 – Bel(A^c)$
    - $Pl(A) \geq Bel(A), \forall A \subseteq \Omega$
    - $Pl(A) \geq P(A) \geq Bel(A)$
  - **Mass-Belief relation** (Moebiïs formula)
    - $m(A) = \sum_{\emptyset \neq B \subset A} (-1)^{(|A|-|B|)} Bel(B),$
  - **Sub-Additive Measure**
    - $Bel(A \cup B) \geq Bel(A) + Bel(B) – Bel(A \cap B)$
    - $Bel(A) + Bel(A^c) \leq 1$
  - **Over-Additive Measure**
    - $Pl(A \cup B) \leq Pl(A) + Pl(B) – Pl(A \cap B)$
    - $Pl(A) + Pl(A^c) \geq 1$

$P(A \cup B) = P(A) + P(B) – P(A \cap B)$

$PA) + P(A^c) = 1$

Belief Functions Theory

- **Basic set functions**
  - **Murder example**

A	∅	{P}	{J}	{P, J}	{M}	{P, M}	{J, M}	Ω
m(A)	0	0	0	0,8	0	0	0	0,2
Bel(A)	0	0	0	0,8	0	0	0	1
Pl(A)	0	1	1	1	0,2	1	1	1

  - **Observe that**

$$\mathrm{Bel}(A \cup B) \geq \mathrm{Bel}(A) + \mathrm{Bel}(B) - \mathrm{Bel}(A \cap B)$$

$$\mathrm{Pl}(A \cup B) \leq \mathrm{Pl}(A) + \mathrm{Pl}(B) - \mathrm{Pl}(A \cap B)$$

  - **Bel et Pl are non additive measures**

Belief Functions Theory

- **Semantics of $m(\emptyset)$**
  - Ω set of possible answers to a question, set of alternatives, ….
  - Closed-world assumption: Ω is exhaustive
    - $m(\emptyset) = 0$: Normal mass
  - Open-world assumption: Ω is not exhaustive
    - Some answers are missed, were inconceivable when modelling the problem
    - $m(\emptyset) > 0$: Transferable Belief Model
    - $m(\emptyset)$ part of belief committed to the assumption $\omega \notin \Omega$
  - From Open-world to Closed-word
    - Redistribution & Normalisation

Belief Functions Theory

- **Belief functions-based model**
  - Information fusion
    - Teledetection
    - Target identification
  - Clustering
  - Databases

Belief Functions Theory

- **In summary belief functions**
  - ☑ More general than probabilistic and possibilistic models
  - ☑ Subjective judgements (agent knowledge)
  - ☑ Modelling both uncertainty and imprecision
  - ☑ Well-modelling of total ignorance
  - ☑ Endowed with a combination rule: Dempster rule
  - ☑ Very interesting when
    - ➢ There is a lack of reliability (Sensors, Testimonies)
    - ➢ Combining heterogeneous information (Multi-sensor fusion, experts knowledge integration, …)

Uncertainty in Databases

Uncertainty modelling

- **Causes of uncertainty**
  - Merging information coming from different sources
  - Lack of knowledge about some information of interest
  - Error in measurements
  - Evolving information
  - ….
- **Two levels of uncertainty**
  - Tuple level
  - Attribute level
    - **The matter of interest**

Uncertain Skyline Queries

- **Principle**

  - Given a set r of n-dimensional tuples

  - A skyline query returns the set of non-dominated tuples in r (in the sense of Pareto optimality)

A tupe t dominates another tuple t' if t is at least as good as t' in all dimensions and strictly better than t' in at least one dimension.

  - The skyline contains the set of the most interesting tuples (even when different and often conflicting criteria are involved in the user query)

  - The elements of the skyline are incomparable

Uncertain Skyline Queries

- **Example**
  - Let r be an extension of relation $\mathcal{Hotel}$
  - Assume that a person is looking for a hotel with good price and near the conference location
  - For instance, t_5 dominates t_1
  - **Skyline = $\mathcal{S}$ = {t_3, t_5, t_2}**
    - **The most interseting hotels**

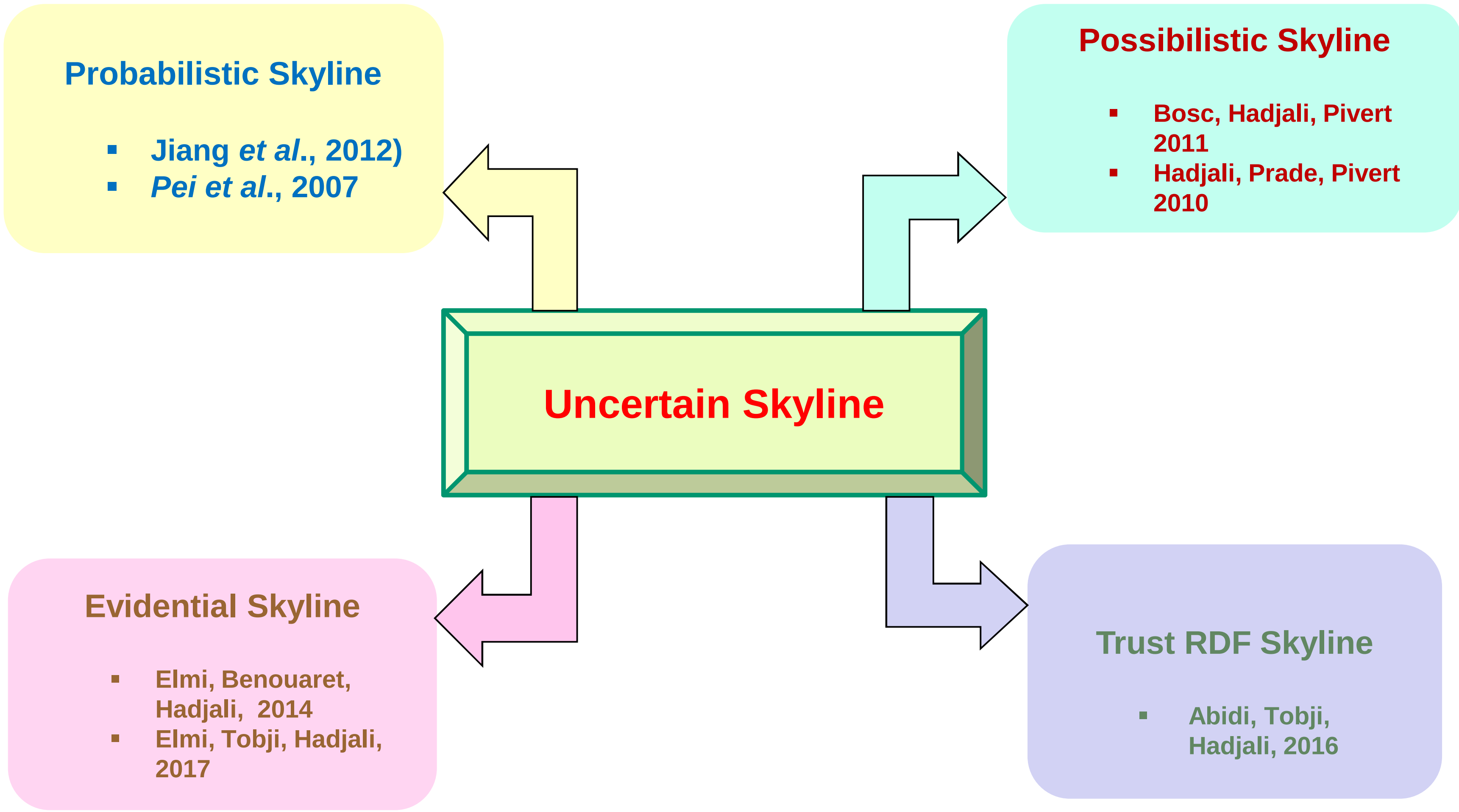
tuple	price	distance	Age	Bedroom num
t_1	4	3	2	2
t_2	5	1	1	2
t_3	1	4	4	1
t_4	2	5	5	1
t_5	2	2	3	1

Uncertain Skyline Queries

- **Very interesting tool**
  - Multicriteria decision making
  - Multidimensional Data Analysis
- **Several extensions**
  - Numerous works done
    - Algorithmic aspect
    - Computational issue
  - Relatively less works
    - Skyline semantics / Uncertain Skyline
    - Recently
      - Fuzzy Skyline (Hadjali, Pivert, Prade, 2011)
      - Skyline relaxation/Refinement (Abbaci et al., 2011)
      - Skyline under uncertainty (Elmi et al., 2014) (Bosc et al., 2011)

Uncertain Skyline

- Let $\mathscr{D}$ be an uncertain Database and Q a Skyline query



Possibilistic Skyline

- **Possibilistic Databases (Bosc, Hadjali, Pivert, 2011)**
  - Values of attributes are modelled thanks to possibilistic distributions

#i	ac	date	loc
i1	{1/a1, 0,6/a2}	{1/d1, 0,7/d2}	c1
i2	{1/a3, 0,6/a4}	d1	c2

  - Possibilistic dominance relationship
  - Possibilistic Skyline

Evidential Skyline

- **Evidential Databases (Elmi et al., 2016, 2014)**
  - Values of attributes are modelled thanks to mass functions

#p	Weight loss	Repayment (%)
p1	<0,1/{16, 18}>, <0,9/{16, 18}>	<0,3/{90}>, <0,7/{90, 100}>
p2	<0,7/{7}>, <0,3/{8, 9}>	<0,8/{70, 80}>, <0,2/{80}>

  - Evidential dominance relationship
  - Evidential Skyline

- **Possible-Words-Based Evidential Skyline: In progress**

Conclusion

Conclusion

- **Each model is insufficient to handle all facets of uncertainty**
  - Some models are more general than others
  - Some are more mature than others
  - Some have interpretations better fitted to a particular situation or problem
  - Some dispose of more convenient tools than others

- **The ultimate aim is**
  - Not to show "which is the best model" but rather
  - When, where, why and how each model should be used?

Thank you

https://www.lias-lab.fr/members/allelhadjali/